As-ifism:
Mathematics and Method Without Metaphysics.
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AIMS
- Carve out an *as-if* interpretation of mathematical structuralism by disentangling *methodological* considerations from *metaphysical* ones.
    - I begin first with Plato to show that much philosophical milk has been spilt owing to our *conflating* the method of mathematics with the method of philosophy.
    - I further use my reading of Plato to develop what I call *as-ifism*, the view that, in mathematics, we treat our hypotheses *as if* they were first principles and we do this with the purpose of solving mathematical problems *not* philosophical ones.
    - I next extend *as-ifism* to modern mathematics wherein the method of mathematics becomes the axiomatic method, noting that this engenders a shift from as-if hypotheses to as-if axioms.
    - Again, I pause to note that the conflation of the method of mathematics with the method of philosophy, witnessed well by the Frege-Hilbert debate, has led to the continued *confusion* of mathematics with metaphysics.
    - Finally, I use a *methodologically* interpreted as-ifism to break Benacerraf's dilemma by showing that there are *two types of existence* at play in mathematics and Science.

PLATO
- Plato kept a clear distinction between mathematics and metaphysics. And the knife he used to slice the difference between the two was method.
- These differences in method *demand* differences in *both* epistemology and ontology
    - The mathematical method requires objects as *objects of thought* and yields *a kind of understanding*.
    - The philosophical method requires objects as of *objects of understanding* and yields *true understanding* or *knowledge itself*.

THE CONFUSION
- The hypothetical method of mathematics is distinct from the metaphysical method of philosophy, and, as such, so is its ontology, and its epistemology.
- Philosophy *as a science* is founded on the dialectical method and the stability of its metaphysical objects, i.e., Forms.
- Mathematics *as a science* is founded on the hypothetical method and the stability of its definitions.

CORRECTING THE CONFUSION
- *The Confusion*: we have continued to conflate the hypothetical method of mathematics with the metaphysical method of philosophy.
- *The Correction*: When I say a mathematical object exists, what I mean is that I treat my hypothesis *as if* it were a first principle and, in so doing, I act *as if* it were tethered to an object.

PLATONIC METHODOLGICAL AS-IFISM
- In mathematics, we treat our hypotheses *as if* they were first principles, and, *consequently*, our objects *as if* they existed, and we do this with the purpose of solving mathematical problems.
- Mathematics as a science is founded on the the hypothetical *method* and the stability of its definitions; it is *not* founded on the DIALECTIC method and the stability of its metaphysical objects.

MODERN AS-IFISM
- This engenders a shift from starting with *as if* hypotheses to starting with *as if* axioms.
- Mathematics as a science is founded on the axiomatic method and the stability of its definitions, now implicitly expressed by the axioms themselves.

THE FREGE HILBERT DEBATE
- For the Fregean axioms-as-first-principles account, the primitive terms employed by the axioms must be defined over a *fixed* domain *before* the statement of the axioms. That is, they must be logically constructed in the case of arithmetic and Kantian constructed in the case of geometry.
- Hilbert, however, takes axioms *as if* they were first principles that themselves implicitly *define* objects, so whatever satisfies the axioms *is* taken as an object that fixes the truth of the axioms.

META-MATHEMATICAL IF-THENISM
- Frege's meta-mathematical account of the method of mathematics was: *if* then axioms are true, *then* this theorem can be justified
- Hilbert's meta-mathematical account of the method of mathematics was: *if* the axioms are consistent, *then* this theorem can be justified.

IF-THENISM VERSUS AS-IFISM
- What I will now consider is whether the standard *logical if-thenist* views can be weakened to the *methodological as-ifist* view that Plato seemed to be offering up

LOGICAL IF-THENISM
- According to the first *deductive if-thenist option,* mathematics is in the business of establishing results in pure logic.
- This first option can either be expressed as "**A ⊃ T**" is logically valid (logically provable) or as the claim that **T** is a logical consequence of **A** (**T** is logically derivable from **A**).

STRUCTURAL IF-THENISM
- On the second *structural if-thenist option,* Frege "views a mathematical theory as studying the properties of all structures satisfying certain defining conditions, but he never makes use of the assumption that such structures exist" [Resnik, 117].

PROBLEMS WITH STRUCTURAL IF-THENISM
- The Structure Problem (Hellman's "Home Address" Problem): the *structural if-thenist* option requires set theory as a *background theory*
- The Consistency Problems (Shapiro's "Turn to Logic or Philosophy" Problem)
- Faced with these problems, Resnik presents us with two alternative routes: Fregean or Carnapian.

METHODOLOGICALLY INTERPRETED STRUCTURAL AS-IFISM
- The basic premise of methodologically interpreted structural as-ifism is that mathematics is used to solve mathematical and physical problems *and* that it is in virtue of these uses that we justified in taking a set of axioms *as if* they were consistent and meta-mathematically justified in taking our background theory *as if* it were true.
- *But*, it is *methodological* considerations, and not metaphysical ones, that "condition" our *as if* assumptions of both the consistency of our mathematical axioms and the truth of our background theory.
- What we get is a "enhanced" version of structural as-ifism, much like Maddy's enhanced if-thenism.
- But we don't analyze the "what follows from what" in terms of a deductivist if-then reading, rather we analyze it in terms of a structuralist as-if reading.
- The proposed *methodologically interpreted structural as-ifist* position is thus even more enhanced:
    - some of our commitments to taking our Axioms as if they were first principles, will be made in light of *mathematical practice*, with the goal of solving mathematical problems,
    - some will be made in light of *mathematical applicability*, with the goal of solving physical problems, and,
    - some will be made in light of *logical/philosophical considerations*, with the goal of solving meta-mathematical problems.

BREAKING BENACERRAF
- We have a shared (Tarskian) semantics: our mathematical statements are made true by reference to mathematical objects.
- We have a reasonable epistemology in mathematics: we come to know mathematical objects via the axiomatic method as objects of thought, we treat them *as if* they exist because we take our axioms *as if* they were True.
- The *difference* is that, in mathematics, existence is a consequence of truth and in science truth is a consequence of existence.

MATHEMATICAL REALISM WITHOUT METAPHYSICS
- In mathematics there is nothing more to existence than *as if* existence, that is, existence on the basis of what we say.
- In science there is more to existence than *as if* existence, there is existence of the basis of what we can show.
- The Problem:
    - By conflating mathematical issues with metaphysical ones, metaphysical realists have made mathematics into a science and so have proclaimed that mathematics must *be about* objects.
    - By conflating metaphysical issues with mathematical ones, structural realists have made science into a language and so have proclaimed that physics has *no objects*.
- The Solution:
    - when we avoid the conflation of mathematical and metaphysical issues, we see that *methodologically* interpreted *structural as-ifism* can be used to provide an account of both the *practice* and the *applicability* of mathematics, *without* reifying mathematical objects and *without* eliminating physical ones!