Why Not Categorical Equivalence?

James Owen Weatherall

Logic and Philosophy of Science
University of California
Irvine, CA USA

Categorical Equivalence Workshop
Munich Center for Mathematical Philosophy
31 July 2018

Apology

I am going to criticize work by (almost) everyone in this room.

Apology

I am going to criticize work by (almost) everyone in this room.

I mean to be constructive—but also provocative.

Apology

I am going to criticize work by (almost) everyone in this room.

I mean to be constructive—but also provocative.

I think work on this topic has been fruitful, but we need to address some foundational problems.

Talk Overview

1. What Have We Done?
2. The Structure of Theories
3. Where do we go from here?

Talk Overview

1. What Have We Done?
2. The Structure of Theories
3. Where do we go from here?

The “official” criterion

Two theories are equivalent if:

1. their categories of models are equivalent; and
2. the functors realizing that equivalence preserve empirical content.

The “official” criterion

Two theories are equivalent if:

1. their categories of models are equivalent; and
2. the functors realizing that equivalence preserve empirical content.

We will return to this, but: (2) has always been left unspecified.

The underlying idea

The idea behind this criterion is that two theories are equivalent if:

The underlying idea

The idea behind this criterion is that two theories are equivalent if:

1. Their mathematical structures are equivalent;

The underlying idea

The idea behind this criterion is that two theories are equivalent if:

1. Their mathematical structures are equivalent;
2. They have the same empirical content; and

The underlying idea

The idea behind this criterion is that two theories are equivalent if:

1. Their mathematical structures are equivalent;
2. They have the same empirical content; and
3. These two equivalences are compatible.

Success in practice

This criterion has been used to argue for the (in)equivalence of several pairs of theories:

Success in practice

This criterion has been used to argue for the (in)equivalence of several pairs of theories:

1. Vector potentials & electromagnetic fields

Success in practice

This criterion has been used to argue for the (in)equivalence of several pairs of theories:

1. Vector potentials & electromagnetic fields
2. Newtonian gravitation & geometrized Newtonian gravitation

Success in practice

This criterion has been used to argue for the (in)equivalence of several pairs of theories:

1. Vector potentials & electromagnetic fields
2. Newtonian gravitation & geometrized Newtonian gravitation
3. Lagrangian mechanics & Hamiltonian mechanics

Success in practice

This criterion has been used to argue for the (in)equivalence of several pairs of theories:

1. Vector potentials & electromagnetic fields
2. Newtonian gravitation & geometrized Newtonian gravitation
3. Lagrangian mechanics & Hamiltonian mechanics
4. Einstein algebras & general relativity

Success in practice

This criterion has been used to argue for the (in)equivalence of several pairs of theories:

1. Vector potentials & electromagnetic fields
2. Newtonian gravitation & geometrized Newtonian gravitation
3. Lagrangian mechanics & Hamiltonian mechanics
4. Einstein algebras & general relativity
5. Various formulations of Yang-Mills theory

I believe categorical equivalence has given the right verdict in all of these cases.

More...

In some cases—EAs & GR; YM theory—I think it has actually taught us something about the theories in question.

More...

In some cases—EAs & GR; YM theory—I think it has actually taught us something about the theories in question.

In others–LM & HM—I think it has provided real insight into differences of interpretation.

More...

In some cases—EAs & GR; YM theory—I think it has actually taught us something about the theories in question.

In others–LM & HM—I think it has provided real insight into differences of interpretation.

And finally, we have seen strong theoretical connections between categorical equivalence and other notions of equivalence, such as definitional/Morita equivalence and sym*.

Some preliminary concerns

Some preliminary concerns

1. Lack of clarity/uniformity about "models" of a physical theory:

Some preliminary concerns

1. Lack of clarity/uniformity about "models" of a physical theory:
   1. A structure representing the complete history of the universe;

Some preliminary concerns

1. Lack of clarity/uniformity about "models" of a physical theory:
   1. A structure representing the complete history of the universe;
   2. A state space with a dynamics specified;

Some preliminary concerns

1. Lack of clarity/uniformity about “models” of a physical theory:
    1. A structure representing the complete history of the universe;
    2. A state space with a dynamics specified;
    3. A state space with observables specified;

Some preliminary concerns

1. Lack of clarity/uniformity about "models" of a physical theory:
   1. A structure representing the complete history of the universe;
   2. A state space with a dynamics specified;
   3. A state space with observables specified;
   4. Something else—such as initial data, a space of equations, etc.

Some preliminary concerns

1. Lack of clarity/uniformity about “models” of a physical theory:
   1. A structure representing the complete history of the universe;
   2. A state space with a dynamics specified;
   3. A state space with observables specified;
   4. Something else—such as initial data, a space of equations, etc.
2. Where are the quantum examples? Stat Mech? Etc.

Some preliminary concerns

1. Lack of clarity/uniformity about “models” of a physical theory:
   1. A structure representing the complete history of the universe;
   2. A state space with a dynamics specified;
   3. A state space with observables specified;
   4. Something else—such as initial data, a space of equations, etc.
2. Where are the quantum examples? Stat Mech? Etc.
3. How does what we are doing relate to “native” applications of categories in physics, such as locally covariant (quantum) field theory, (higher) gauge theory, fusion categories and anyons, etc.

Some preliminary concerns

1. Lack of clarity/uniformity about “models” of a physical theory:
    1. A structure representing the complete history of the universe;
    2. A state space with a dynamics specified;
    3. A state space with observables specified;
    4. Something else—such as initial data, a space of equations, etc.
2. Where are the quantum examples? Stat Mech? Etc.
3. How does what we are doing relate to “native” applications of categories in physics, such as locally covariant (quantum) field theory, (higher) gauge theory, fusion categories and anyons, etc.
4. Can we establish “empirical equivalence” once and for all? How do we capture the modal character of realistic cases?

Some preliminary concerns

1. Lack of clarity/uniformity about "models" of a physical theory:
   1. A structure representing the complete history of the universe;
   2. A state space with a dynamics specified;
   3. A state space with observables specified;
   4. Something else—such as initial data, a space of equations, etc.
2. Where are the quantum examples? Stat Mech? Etc.
3. How does what we are doing relate to "native" applications of categories in physics, such as locally covariant (quantum) field theory, (higher) gauge theory, fusion categories and anyons, etc.
4. Can we establish "empirical equivalence" once and for all? How do we capture the modal character of realistic cases?
5. Physical theories are messy affairs including all sorts of arguments, numerical methods, biases, etc. Is there a single category associated with a physical theory?

Talk Overview

1. What Have We Done?
2. The Structure of Theories
3. Where do we go from here?

Confession

Confession

I do not think that a “category (of models?)” (necessarily) captures the “structure” of a theory.

Ideology

First: what is “category structure”?

Ideology

First: what is “category structure”?

To understand what structure a mathematical gadget has, one should study the maps that preserve that structure.

Ideology

First: what is “category structure”?

To understand what structure a mathematical gadget has, one should study the maps that preserve that structure.

Eg. groups are collections of elements distinguished (only) by their multiplicative relations with other group elements.

Ideology

(n-)Categorical (n-)equivalence preserves (n-)category structure.

Ideology

(n-)Categorical (n-)equivalence preserves (n-)category structure.

Thus we should reflect on what is preserved by categorical equivalence.

Ideology

(n-)Categorical (n-)equivalence preserves (n-)category structure.

Thus we should reflect on what is preserved by categorical equivalence.

In analogy to groups: a category is a collection of objects distinguished (only, and only up to isomorphism) by their arrow-algebraic relations with other objects.

Ideology

In other words: the arrows carry all the information; objects are basically placeholders.

Ideology

In other words: the arrows carry all the information; objects are basically placeholders.

(This isn't meant to be a surprise!)

Ideology

The "internal structure" of objects is not preserved under categorical equivalence.

Ideology

The “internal structure” of objects is not preserved under categorical equivalence.

(Recall the classic trivialization concern!)

Ideology

The “internal structure” of objects is not preserved under categorical equivalence.

(Recall the classic trivialization concern!)

Thus, to refer to that structure in reasoning or constructions involving categories is [insert adjective suggesting disapproval].

Ideology

The “internal structure” of objects is not preserved under categorical equivalence.

(Recall the classic trivialization concern!)

Thus, to refer to that structure in reasoning or constructions involving categories is [insert adjective suggesting disapproval].

Analogy: there are no theorems of differential geometry that rely on the points being Dedekind cuts, rather than Cauchy sequences.

Question

Can pure “category structure” capture a theory?

Answer

Sometimes, and sort of.

Answer

Consider the category of sets, **Set**.

Answer

Consider the category of sets, **Set.** (Of course, by the above, that the objects are sets is immaterial.)

Answer

Consider the category of sets, **Set**. (Of course, by the above, that the objects are sets is immaterial.)

Using only arrow constructions (basically, particular arrows and limits), we can reason about sets in detail.

Answer

Consider the category of sets, **Set**. (Of course, by the above, that the objects are sets is immaterial.)

Using only arrow constructions (basically, particular arrows and limits), we can reason about sets in detail.

Eg. maps from the terminal object are elements of sets; monics are subsets; coproducts are disjoint unions, etc.

Answer

Consider the category of sets, **Set**. (Of course, by the above, that the objects are sets is immaterial.)

Using only arrow constructions (basically, particular arrows and limits), we can reason about sets in detail.

Eg. maps from the terminal object are elements of sets; monics are subsets; coproducts are disjoint unions, etc.

Sets are distinguished, up to isomorphism (bijection), by the arrows of **Set**; non-isomorphic sets sit in different positions in the graph of arrows.

Abstracting

Is this true for all categories?

Abstracting

Is this true for all categories? **No.**

Abstracting

Proposal: A theory is captured by category structure only if the arrows of that category can distinguish the objects, up to isomorphism.

Abstracting

Proposal: A theory is captured by category structure only if the arrows of that category can distinguish the objects, up to isomorphism.

This often fails.

Example

Consider General relativity.

Example

Consider General relativity.

Let **GR** be a category whose objects are relativistic spacetimes and whose arrows are isometries.

Example

Consider General relativity.

Let **GR** be a category whose objects are relativistic spacetimes and whose arrows are isometries.

Then objects with no symmetries are not distinguished from one another.

Example

Consider General relativity.

Let **GR** be a category whose objects are relativistic spacetimes and whose arrows are isometries.

Then objects with no symmetries are not distinguished from one another.

(Question: does adding isometric embeddings help?

Example

Consider General relativity.

Let **GR** be a category whose objects are relativistic spacetimes and whose arrows are isometries.

Then objects with no symmetries are not distinguished from one another.

(Question: does adding isometric embeddings help? I doubt it!)

The Geroch property

A category C has the **Geroch property** if every full, faithful, and essentially surjective functor $F : C \to C$ is naturally isomorphic to 1_C.

GR revisited

GR does not have the Geroch property.

The Geroch property revisited

The Geroch property revisited

The Geroch property is (probably) neither necessary nor sufficient for a category to capture the “structure” of a theory.

Example [sufficiency]

Consider the theory “Directions”.

Example [sufficiency]

Consider the theory “Directions”.

Directions says “the cardinal directions form a two dimensional vector space, with ‘north’ and ‘east’ physically distinguished”.

Example [sufficiency]

Consider the theory “Directions”.

Directions says “the cardinal directions form a two dimensional vector space, with ‘north’ and ‘east’ physically distinguished”.

Its category of models, **Di**, has, as objects, two dimensional vector spaces with ordered basis, and as arrows, linear bijections that preserve that basis.

Example [sufficiency]

Consider the theory "Baubles".

Example [sufficiency]

Consider the theory "Baubles".

Baubles says "there are two shiny things, one of which is red and the other of which is blue".

Example [sufficiency]

Consider the theory “Baubles”.

Baubles says “there are two shiny things, one of which is red and the other of which is blue”.

Its category of models, **Bau**, has, as objects, ordered pairs, and as arrows, bijections that preserve order.

Example [sufficiency]

Both **Bau** and **Di** satisfy the Geroch property.

Example [sufficiency]

Both **Bau** and **Di** satisfy the Geroch property.

But it is hard to see how either captures the structure of their respective theories.

Example [sufficiency]

Both **Bau** and **Di** satisfy the Geroch property.

But it is hard to see how either captures the structure of their respective theories.

Indeed, **Di** and **Bau** are equivalent, despite the models having very different internal structures.

Example [sufficiency]

If the **Bau-Di** equivalence seems trivial, consider instead...

Example [sufficiency]

If the **Bau-Di** equivalence seems trivial, consider instead...

ManDi: objects are finite dimensional vector spaces with preferred ordered bases; arrows are linear bijections preserving basis.

Example [sufficiency]

If the **Bau-Di** equivalence seems trivial, consider instead...

ManDi: objects are finite dimensional vector spaces with preferred ordered bases; arrows are linear bijections preserving basis.

BauMo: objects are ordered (finite) n-tuples; arrows are bijections preserving order.

Example [sufficiency]

If the **Bau-Di** equivalence seems trivial, consider instead...

ManDi: objects are finite dimensional vector spaces with preferred ordered bases; arrows are linear bijections preserving basis.

BauMo: objects are ordered (finite) n-tuples; arrows are bijections preserving order.

We can even add "embeddings" in these cases, without violating the Geroch property!

Example [necessity]

Consider the category **Ring** whose objects are rings and whose arrows are ring homomorphisms.

Example [necessity]

Consider the category **Ring** whose objects are rings and whose arrows are ring homomorphisms.

There is an endoequivalence *Op* that takes rings to their opposite rings, which is not naturally isomorphic to the identity.

Example [necessity]

Consider the category **Ring** whose objects are rings and whose arrows are ring homomorphisms.

There is an endoequivalence *Op* that takes rings to their opposite rings, which is not naturally isomorphic to the identity.

But this endoequivalence captures a real "symmetry" of the theory of rings.

Example [necessity]

Consider the category **Ring** whose objects are rings and whose arrows are ring homomorphisms.

There is an endoequivalence *Op* that takes rings to their opposite rings, which is not naturally isomorphic to the identity.

But this endoequivalence captures a real "symmetry" of the theory of rings.

I do not think this shows that **Ring** does not capture the structure of rings.

No necessity?

If the Geroch property is neither necessary nor sufficient, then who cares?

No necessity?

If the Geroch property is neither necessary nor sufficient, then who cares?

The **Ring** case is different from **GR**. The functor *Op* captures a feature of ring structure.

No necessity?

If the Geroch property is neither necessary nor sufficient, then who cares?

The **Ring** case is different from **GR**. The functor *Op* captures a feature of ring structure.

Analogous functors do not such thing for **GR**.

No necessity?

If the Geroch property is neither necessary nor sufficient, then who cares?

The **Ring** case is different from **GR**. The functor *Op* captures a feature of ring structure.

Analogous functors do not such thing for **GR**.

This suggests the Geroch property is not quite what we want.

No necessity?

If the Geroch property is neither necessary nor sufficient, then who cares?

The **Ring** case is different from **GR**. The functor *Op* captures a feature of ring structure.

Analogous functors do not such thing for **GR**.

This suggests the Geroch property is not quite what we want. **But the instinct seems right!**

Questions

Is there a property that captures the intuition behind the Geroch property better?

Questions

Is there a property that captures the intuition behind the Geroch property better?

What features do **Set**, **Group**, **Ring**, etc. share that **GR** lack?

Questions

Is there a property that captures the intuition behind the Geroch property better?

What features do **Set**, **Group**, **Ring**, etc. share that **GR** lack?

Do all "natural" "concrete" categories (such as **Man**) share these features?

Questions

Is there a property that captures the intuition behind the Geroch property better?

What features do **Set**, **Group**, **Ring**, etc. share that **GR** lack?

Do all "natural" "concrete" categories (such as **Man**) share these features?

Does *any* physical theory's category of models?

Talk Overview

1. What Have We Done?
2. The Structure of Theories
3. Where do we go from here?

The Mines of Morita

I see three ways of responding to the situation above.

Option 1: The Parnsip Theory

Option 1: The Parnsip Theory

We could find a good Geroch-like property.

Option 1: The Parnsip Theory

We could find a good Geroch-like property.

Then categorical equivalence yields theoretical equivalence only for suitably Geroch-like theories.

Option 1: The Parnsip Theory

We could find a good Geroch-like property.

Then categorical equivalence yields theoretical equivalence only for suitably Geroch-like theories.

This option has two problems.

Option 1: The Parnsip Theory

We could find a good Geroch-like property.

Then categorical equivalence yields theoretical equivalence only for suitably Geroch-like theories.

This option has two problems.

1. What is the property?

Option 1: The Parnsip Theory

We could find a good Geroch-like property.

Then categorical equivalence yields theoretical equivalence only for suitably Geroch-like theories.

This option has two problems.

1. What is the property?
2. Do we believe any “theories in the wild” have this property?

Option 2: The Beet Theory

Option 2: The Beet Theory

A second option is to change the criterion of equivalence, à la Hudetz's DCE or Barrett's well-behaved functors.

Option 2: The Beet Theory

A second option is to change the criterion of equivalence, à la Hudetz's DCE or Barrett's well-behaved functors.

Indeed, bad failures of the Geroch property (e.g. **GR**) are arguably failures of “definability” of endoequivalences.

Beet It

Not so fast.

Beet It

Recall the ideology above.

Beet It

Recall the ideology above.

By changing the “structure preserving” maps we consider, we are implicitly changing the structures preserved by those maps.

Beet It

Recall the ideology above.

By changing the “structure preserving” maps we consider, we are implicitly changing the structures preserved by those maps.

It is analogous to noting that not all functions are well-behaved, and then restricting attention to continuous or smooth maps.

We've Got the Beet

This can be done, but we should be explicit:

We’ve Got the Beet

This can be done, but we should be explicit:

We should introduce a new kind of structure, a **Hudetz category**; Hudetz category structure is preserved by definable functors.

We've Got the Beet

This can be done, but we should be explicit:

We should introduce a new kind of structure, a **Hudetz category**; Hudetz category structure is preserved by definable functors.

But what is a Hudetz category?

We’ve Got the Beet

This can be done, but we should be explicit:

We should introduce a new kind of structure, a **Hudetz category**; Hudetz category structure is preserved by definable functors.

But what is a Hudetz category?

Possible analogues: tangent bundle (a vector bundle + solder form); vector space with basis; a manifold of states (?).

Possible slippage?

Theories in the wild can, arguably, be associated with categories.

Possible slippage?

Theories in the wild can, arguably, be associated with categories.

But can they be associated with Hudetz categories?

Possible slippage?

Theories in the wild can, arguably, be associated with categories.

But can they be associated with Hudetz categories?

If so, much of the work will be in identifying the n-th order theory.

Possible slippage?

Theories in the wild can, arguably, be associated with categories.

But can they be associated with Hudetz categories?

If so, much of the work will be in identifying the n-th order theory.

Question: What role are categories playing in this?

Sugarbeet Theory

Re: Thomas's point in response to Laurenz an hour or two ago:

Sugarbeet Theory

Re: Thomas's point in response to Laurenz an hour or two ago:

Consider the following analogy: a "well-behaved functor" is a bit like a "continuous function".

Sugarbeet Theory

Re: Thomas's point in response to Laurenz an hour or two ago:

Consider the following analogy: a "well-behaved functor" is a bit like a "continuous function".

Of course, we need a topology to make sense of continuous functions.

Sugarbeet Theory

Re: Thomas's point in response to Laurenz an hour or two ago:

Consider the following analogy: a "well-behaved functor" is a bit like a "continuous function".

Of course, we need a topology to make sense of continuous functions.

But! All polynomials are continuous in a particular canonical topology on $\mathbb{R}$.

Sugarbeet Theory

Re: Thomas's point in response to Laurenz an hour or two ago:

Consider the following analogy: a "well-behaved functor" is a bit like a "continuous function".

Of course, we need a topology to make sense of continuous functions.

But! All polynomials are continuous in a particular canonical topology on $\mathbb{R}$.

Perhaps the functors we usually see are like this.

Option 3: The Potato Theory

Option 3: The Potato Theory

Let's return to the examples of "successes" mentioned above.

Option 3: The Potato Theory

Let's return to the examples of "successes" mentioned above.

1. Vector potentials & electromagnetic fields
2. Newtonian gravitation & geometrized Newtonian gravitation
3. Lagrangian mechanics & Hamiltonian mechanics
4. Einstein algebras & general relativity
5. Various formulations of Yang-Mills theory

Option 3: The Potato Theory

Let's return to the examples of "successes" mentioned above.

1. Vector potentials & electromagnetic fields
2. Newtonian gravitation & geometrized Newtonian gravitation
3. Lagrangian mechanics & Hamiltonian mechanics
4. Einstein algebras & general relativity
5. Various formulations of Yang-Mills theory

Why do these work?

Potatoes have all 9 essential amino acids

In each case, some (deep?) mathematical fact is used.

Potatoes have all 9 essential amino acids

In each case, some (deep?) mathematical fact is used.

1. Vector potentials & electromagnetic fields
2. Newtonian gravitation & GNG
3. Lagrangian & Hamiltonian mechanics
4. Einstein algebras & general relativity
5. Various formulations of Yang-Mills theory

Potatoes have all 9 essential amino acids

In each case, some (deep?) mathematical fact is used.

1. Vector potentials & electromagnetic fields (Poincaré's lemma)
2. Newtonian gravitation & GNG
3. Lagrangian & Hamiltonian mechanics
4. Einstein algebras & general relativity
5. Various formulations of Yang-Mills theory

Potatoes have all 9 essential amino acids

In each case, some (deep?) mathematical fact is used.

1. Vector potentials & electromagnetic fields (Poincaré's lemma)
2. Newtonian gravitation & GNG (Trautman theorems)
3. Lagrangian & Hamiltonian mechanics
4. Einstein algebras & general relativity
5. Various formulations of Yang-Mills theory

Potatoes have all 9 essential amino acids

In each case, some (deep?) mathematical fact is used.

1. Vector potentials & electromagnetic fields (Poincaré's lemma)
2. Newtonian gravitation & GNG (Trautman theorems)
3. Lagrangian & Hamiltonian mechanics (Legendre transformation)
4. Einstein algebras & general relativity
5. Various formulations of Yang-Mills theory

Potatoes have all 9 essential amino acids

In each case, some (deep?) mathematical fact is used.

1. Vector potentials & electromagnetic fields (Poincaré's lemma)
2. Newtonian gravitation & GNG (Trautman theorems)
3. Lagrangian & Hamiltonian mechanics (Legendre transformation)
4. Einstein algebras & general relativity (space-function duality)
5. Various formulations of Yang-Mills theory

Potatoes have all 9 essential amino acids

In each case, some (deep?) mathematical fact is used.

1. Vector potentials & electromagnetic fields (Poincaré's lemma)
2. Newtonian gravitation & GNG (Trautman theorems)
3. Lagrangian & Hamiltonian mechanics (Legendre transformation)
4. Einstein algebras & general relativity (space-function duality)
5. Various formulations of Yang-Mills theory (Barrett recovery)

Potatoes have all 9 essential amino acids

In each case, some (deep?) mathematical fact is used.

1. Vector potentials & electromagnetic fields (Poincaré's lemma)
2. Newtonian gravitation & GNG (Trautman theorems)
3. Lagrangian & Hamiltonian mechanics (Legendre transformation)
4. Einstein algebras & general relativity (space-function duality)
5. Various formulations of Yang-Mills theory (Barrett recovery)

We then show that these relationships are functorial.

What We Talk about When We Talk About Equivalence

Recall what we are really showing:

What We Talk about When We Talk About Equivalence

Recall what we are really showing:

1. The mapping on objects takes every model of each theory to an essentially unique model of the other theory;

What We Talk about When We Talk About Equivalence

Recall what we are really showing:

1. The mapping on objects takes every model of each theory to an essentially unique model of the other theory;
2. This mapping is such that every structure-preserving map between the models of one theory corresponds uniquely to a structure-preserving map between the corresponding models of the other theory, and vice versa.

What We Talk about When We Talk About Equivalence

Recall what we are really showing:

1. The mapping on objects takes every model of each theory to an essentially unique model of the other theory;
2. This mapping is such that every structure-preserving map between the models of one theory corresponds uniquely to a structure-preserving map between the corresponding models of the other theory, and vice versa.
3. In particular, symmetries are preserved when we pass between theories.

What We Talk about When We Talk About Equivalence

Recall what we are really showing:

1. The mapping on objects takes every model of each theory to an essentially unique model of the other theory;
2. This mapping is such that every structure-preserving map between the models of one theory corresponds uniquely to a structure-preserving map between the corresponding models of the other theory, and vice versa.
3. In particular, symmetries are preserved when we pass between theories.

These are natural things to (try to) establish about any mathematical relationship.

Rosenstock's heuristic

This suggests that what we are really doing is **abstracting** "pure category" structure from a richer characterization of theories.

Rosenstock's heuristic

This suggests that what we are really doing is **abstracting** "pure category" structure from a richer characterization of theories.

From this perspective, categorical equivalence is not sufficient for equivalence; rather, it is necessary.

Rosenstock's heuristic

This suggests that what we are really doing is **abstracting** "pure category" structure from a richer characterization of theories.

From this perspective, categorical equivalence is not sufficient for equivalence; rather, it is necessary.

Pessimistic gloss: All of the real work is done by deep, but non-categorical, mathematics that establishes **empirical equivalence**; category theory is just window dressing!

The end

Thank you!